Algebra 2
Chapter 7

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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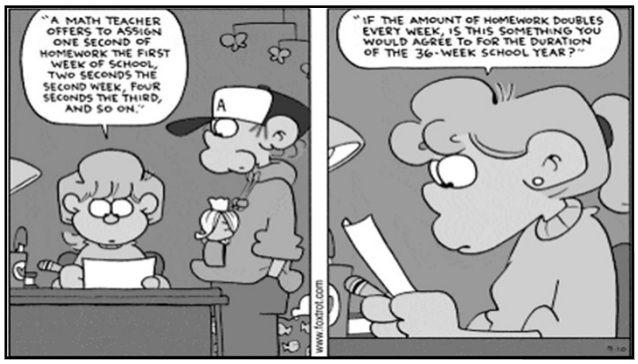
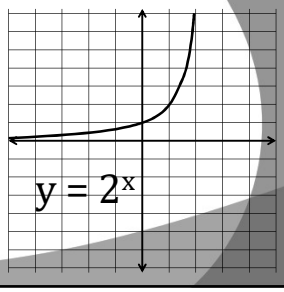
7.1 Graph Exponential Growth Functions



7.1 Graph Exponential Growth Functions

- Exponential Function
  - $y = b^x$
  - Base (b) is a positive number other than 1



7.1 Graph Exponential Growth Functions

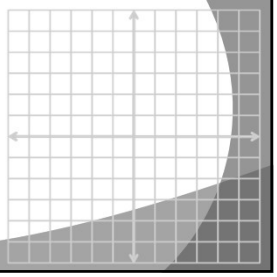
- $y = a \cdot 2^x$
  - y-intercept = a
  - x-axis is the asymptote of graph

7.1 Graph Exponential Growth Functions

- Exponential Growth Function
  - $y = a \cdot b^{x-h} + k$
- To graph
  - Start with $y = b^x$
  - Multiply y-coordinates by a
  - Move up k and right h
  - (or make table of values)
- Properties of the graph
- y-intercept = a (if h and k=0)
- y = k is asymptote
- Domain is all real numbers
- Range
  - y > k if a > 0
  - y < k if a < 0

7.1 Graph Exponential Growth Functions

- Graph
- $y = 3 \cdot 2^{x-3} - 2$



7.1 Graph Exponential Growth Functions

- Exponential Growth Model (word problems)
  - $y = a(1 + r)^t$
    - y = current amount
    - a = initial amount
    - r = growth percent
    - 1 + r = growth factor
    - t = time

7.1 Graph Exponential Growth Functions

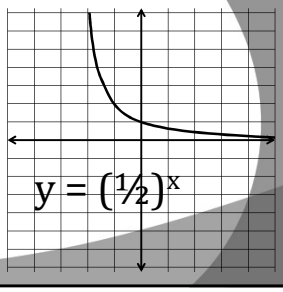
- Compound Interest
- $A = P\left(1 + \frac{r}{n}\right)^{nt}$
  - A = current amount
  - P = principle (initial amount)
  - r = percentage rate
  - n = number of times compounded per year
  - t = time in years

7.1 Graph Exponential Growth Functions

- If you put $200 into a CD (Certificate of Deposit) that earns 4% interest, how much money will you have after 2 years if you compound the interest monthly? daily?

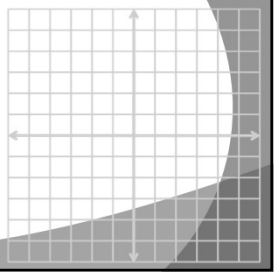
7.2 Graph Exponential Decay Functions

- Exponential Decay
  - $y = a \cdot b^x$
  - $a > 0$
  - $0 < b < 1$
- Follows same rules as growth
  - y-intercept = a
  - y = k is asymptote
  - $y = a \cdot b^{x-h} + k$



7.2 Graph Exponential Decay Functions

- Graph
- $y = 2 \cdot (½)^{x+3} - 2$



7.2 Graph Exponential Decay Functions

- Exponential Decay Model (word problems)
  - $y = a(1 - r)^t$
    - y = current amount
    - a = initial amount
    - r = decay percent
    - 1 – r = decay factor
    - t = time

7.2 Graph Exponential Decay Functions

- A new car cost $23000. The value decreases by 15% each year. Write a model of this decay. How much will the car be worth in 5 years? 10 years?

7.3 Use Functions Involving *e*

- In math, there are some special numbers like π or *i*
- Today we will learn about *e*

7.3 Use Functions Involving *e*

- *e*
  - Called the natural base
  - Named after Leonard Euler who discovered it
    - (Pronounced "oil-er")
  - Found by putting really big numbers into $\left(1+\frac{1}{n}\right)^n$ = 2.718281828459…
  - Irrational number like π

7.3 Use Functions Involving *e*

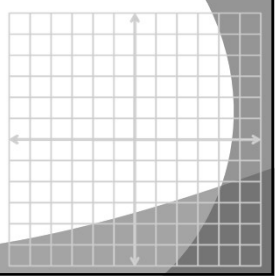
- Simplifying natural base expressions
  - Just treat *e* like a regular variable
- $\frac{24e^8}{8e^5}$
- $(2e^{-5x})^-$

7.3 Use Functions Involving *e*

- Evaluate the natural base expressions using your calculator
- e^3
- $e^{-0.12}$

7.3 Use Functions Involving *e*

- To graph make a table of values
- $f(x) = a \cdot e^{rx}$
  - $a > 0$
  - If $r > 0$ → growth
  - If $r < 0$ → decay
- Graph $y = 2e^{0.5x}$



7.3 Use Functions Involving *e*

- Compound Interest
- $A = P\left(1 + \frac{r}{n}\right)^{nt}$
  - A = current amount
  - P = principle (initial amount)
  - r = percentage rate
  - n = number of times compounded per year
  - t = time in years
- Compounded continuously
  - $A = Pe^{rt}$

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Definition of Logarithm with Base b
- $\log_b y = x \iff b^x = y$
- Read as "log base b of y equals x"
- Rewriting logarithmic equations
- $\log_3 9 = 2 \rightarrow$
- $\log_8 1 = 0 \rightarrow$
- $\log_5(1 / 25) = -2 \rightarrow$

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Special Logs
  - $\log_b 1 = 0$
  - $\log_b b = 1$
- Evaluate
  - $\log_4 64$
  - $\log_2 \frac{1}{8}$
  - $\log_{1/4} 256$

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Using a calculator
- Common Log (base 10)
  - $\log_{10} x = \log x$
  - Find log 12
- Natural Log (base e)
  - $\log_e x = \ln x$
  - Find ln 2

7.4 Evaluate Logarithms and Graph Logarithmic Functions

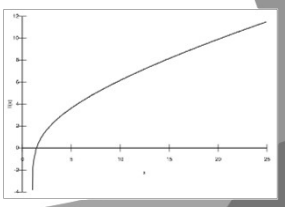
- When the bases are the same, the base and the log cancel
- $5^{\log_5 7} = 7$
- $\log_3 81^x$
- $= \log_3 3^{4x}$
- $= 4x$

7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Finding Inverses of Logs
- $y = \log_8 x$
- $x = \log_8 y$ Switch x and y
- $y = 8^x$ Rewrite to solve for y
- To graph logs
  - Find the inverse
  - Make a table of values for the inverse
  - Graph the log by switching the x and y coordinates of the inverse.

7.4 Evaluate Logarithms and Graph Logarithmic Functions

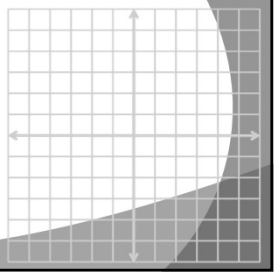
- Properties of graphs of logs
- $y = \log_b (x - h) + k$
  - x = h is vert. asymptote
  - Domain is x > h
  - Range is all real numbers
  - If b > 1, graph rises
  - If 0 < b < 1, graph falls



7.4 Evaluate Logarithms and Graph Logarithmic Functions

- Graph
  - $y = \log_2 x$
  - Inverse
  - $x = \log_2 y$
  - $y = 2^x$

x	y
-3	1/8
-2	¼
-1	½
0	1
1	2
2	4
3	8



7.5 Apply Properties of Logarithms

- Product Property
  - $\log_b uv = \log_b u + \log_b v$
- Quotient Property
  - $\log_b \frac{u}{v} = \log_b u - \log_b v$
- Power Property
  - $\log_b u^n = n \log_b u$

7.5 Apply Properties of Logarithms

- Use $\log_9 5 = 0.732$ and $\log_9 11 = 1.091$ to find
  - $\log_9 \frac{5}{11}$
  - $\log_9 55$
  - $\log_9 25$

7.5 Apply Properties of Logarithms

- Expand: $\log_5 2x^6$
- Condense: $2 \log_3 7 - 5 \log_3 x$

7.5 Apply Properties of Logarithms

- Change-of-Base Formula
  - $\log_c u = \frac{\log_b u}{\log_b c}$
- Evaluate $\log_4 8$

7.6 Solve Exponential and Logarithmic Equations

- Solving Exponential Equations
  - Method 1) if the bases are equal, then exponents are equal
  - $2^{4x} = 32^{x-1}$

7.6 Solve Exponential and Logarithmic Equations

- Solving Exponential Equations
  - Method 2) take log of both sides
  - $4^x = 15$
- $5^{x+2} + 3 = 25$

7.6 Solve Exponential and Logarithmic Equations

- Solving Logarithmic Equations
  - Method 1) if the bases are equal, then logs are equal
  - $\log_3 (5x - 1) = \log_3 (x + 7)$

7.6 Solve Exponential and Logarithmic Equations

- Solving Logarithmic Equations
  - Method 2) exponentiating both sides
    - Make both sides exponents with the base of the log
  - $\log_4 (x + 3) = 2$

7.6 Solve Exponential and Logarithmic Equations

- $\log_2 2x + \log_2(x - 3) = 3$

7.7 Write and Apply Exponential and Power Functions

- Just as 2 points determine a line, so 2 points will determine an exponential equation.

7.7 Write and Apply Exponential and Power Functions

- Exponential Function
  - $y = a\,b^x$
- If given 2 points
  - Fill in both points to get two equations
  - Solve for a and b by substitution

7.7 Write and Apply Exponential and Power Functions

- Find the exponential function that goes through (-1, 0.0625) and (2, 32)

7.7 Write and Apply Exponential and Power Functions

- Steps if given a table of values
  - Find ln y of all points
  - Graph ln y vs x
  - Draw the best fit straight line
  - Pick two points on the line and find equation of line (remember to use ln y instead of just y)
  - Solve for y
- OR use the ExpReg feature on a graphing calculator
  - Enter points in STAT → EDIT
  - Go to STAT → CALC → ExpReg → Enter → Enter

7.7 Write and Apply Exponential and Power Functions

- Writing a Power Function
  - $y = a x^b$
- Steps are the same as for exponential function
  - Fill in both points to get two equations
  - Solve for a and b by substitution

7.7 Write and Apply Exponential and Power Functions

- Write power function through (3, 8) and (9, 12)

7.7 Write and Apply Exponential and Power Functions

- Steps if given a table of values
  - Find ln y and ln x of all points
  - Graph ln y vs ln x
  - Draw the best fit straight line
  - Pick two points on the line and find equation of line (remember to use ln y and ln x instead of just y)
  - Solve for y
- OR use the PwrReg feature on a graphing calculator
  - Enter points in STAT → EDIT
  - Go to STAT → CALC → PwrReg → Enter → Enter